A Non-Fickian Mixing Model for Stratified Turbulent Flows

Tamay M. Özgökmen
Division of Meteorology and Physical Oceanography
Rosenstiel School of Marine and Atmospheric Science
4600 Rickenbacker Causeway
Miami, Florida 33149
phone: 305 421 4053, fax: 305 421 4696, email: tozgokmen@rsmas.miami.edu

Award #: N00014-09-1-0267
http://www.rsmas.miami.edu/personal/tamay/3D/mli.html/

LONG-TERM GOALS

The long term goal of this project is to develop a better understanding of oceanic processes in the range of 100 m to 10 km, in the so-called sub-meso-scale range. In particular, it is important to explore and find out whether and what type of sub-meso-scale instabilities exist, how they are connected to both larger scale and smaller scale motions, and to what extent they influence transport processes in the ocean. Another important objective of this project is to test how well subgrid-scale (SGS) models for large eddy simulations (LES) work in the presence of backward energy cascade that may be characteristic in sub-meso-scale motions. Another long term objective of this effort would be how to improve the predictive skill of the Navy numerical models in the light of the impact of sub-meso-scale motions in the ocean.

OBJECTIVES

My main objective in this first phase of the project is to model mixed layer instabilities, investigate their behavior and try to develop sampling strategies using synthetic drifters and tracers, especially considering potential limitations of resources during the upcoming experimental phase of the Lateral Mixing DRI.

APPROACH

The work is based on large eddy simulations using the non-hydrostatic spectral element model Nek5000 (Fischer, 1997).

WORK COMPLETED

A set of 40 LES experiments are conducted in order to develop a better understanding of the physical characteristics of mixed layer instabilities, to test various sampling strategies and explore mixing and relative dispersion induced by these fields.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **2009**	2. REPORT TYPE	3. DATES COVERED **00-00-2009 to 00-00-2009**

4. TITLE AND SUBTITLE **A Non-Fickian Mixing Model for Stratified Turbulent Flows**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Rosenstiel School of Marine and Atmospheric Science,Division of Meteorology and Physical Oceanography,4600 Rickenbacker Causeway,Miami,FL,33149**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**Same as Report (SAR)**	**8**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

RESULTS

1) Model configuration and generic evolution of mixed-layer instability:

LES approach is taken in light of the small scale motions potentially exhibited by mixed layer instabilities and the vertical accelerations that are large enough to violate the hydrostatic approximation. A number of SGS models have been tested for stratified mixing problems using the spectral element model Nek5000 (Özgökmen et al., 2009a,b). This spectral element model combines the geometrical flexibility of finite element models with the numerical accuracy of spectral expansion. While the geometrical flexibility of the model is not yet important for the problems considered thus far, numerical accuracy is important not only to minimize dissipation and dispersion errors, but also to accurately test SGS models with minimal interference from numerical dissipation. LES of a mixed-layer instability problem is aimed as a test bed for developing tracer and drifter sampling strategies.

Table 1: Computational domain and discretization in LES experiments

Domain: $L_x = L_y = 10\,\mathrm{km}$, $H = 500$ m

Exp	spatial points	$\Delta x = \Delta y$	Δz	Wall Clock Time
standart	4,293,185	16 to 57 m	0.6 to 29 m	$\sim$ 50 hours on 64 processors
high-res	27,994,681	5 to 32 m	0.2 to 16 m	$\sim$ 50 hours on 256 processors

The model domain is a box with dimensions of 10 km by 10 km in the horizontal and 500 m in the vertical directions. It is initialized by mixed layers that are 50 to 100 m deep and separated by 2 to 4 km wide fronts. The model is run on a coarse grid to attain a (near) geostrophic balance. Then a finer mesh is used to simulate the instabilities. In most experiments, the grid spacing of the mesh changes between 16 to 57 m in the horizontal, and from 0.6 m near the surface to 29 m at the bottom. Overall, approximately 4.3 million grid points are used (a medium scale computation). The total integration period corresponds to a real time of 100 days, which takes 50 hours of wall-clock time on 64 processors of an IBM P575 system. Higher resolution simulations with up to 28 million grid points have also been conducted (Table 1).

Snaphots of the density perturbation field defining the evolution of the front are shown in Fig. 1. More than two dozen simulations are conducted with different stratification, rotation parameters, mixed-layer depths and front thicknesses. The story remains very robust; the front is always unstable, initially to inertial instability, which leads to interleaving transporting light water to rightward and dense water to leftward. Dense water intrusions are gravitationally unstable leading to symmetric instability and small-scale motions. The existence of backward energy transfer is evident from the evolution of small scale features toward larger scale eddies. During these initial phases, the instability is confined to the density interface and the far field remains unperturbed. In the next stage, the exchange flow acts to destroy the density interface. The exchange takes place by two mechanisms: by surface jets propagating in the opposite direction in between the surface eddies, and other perturbations at the bottom of the warmer (left) mixed layer, which are probably not detectible at the surface. The surface signature of the instability consists of mostly anticyclonic eddies that propagate rightward.

2) Sensitivities to numerical dissipation and initial density gradient:

The sensitivity of the results to numerical dissipation is explored by changing the Reynolds and Peclet numbers. The initial interleaving in the mixed layer instability shows a significantly slower growth rate when numerical dissipation is high, namely when the Reynolds number is 10^4. For Reynolds number larger by 10 or 100 fold, no major difference is observed (Fig. 2).

A number of experiments are conducted to explore the behavior as a function of the initial density gradient, as quantified by the Froude number. When the Froude number is small (a large density jump across the front), we see formation of coherent eddies that appear to stir but not mix. On the other hand, weak fronts characterized by large Froude numbers exhibit much smaller scale features (Fig. 3). In order to quantify mixing, which is the irreversible modification of water mass distributions, the background/reference potential energy (RPE) is computed, because RPE exactly quantifies mixing in an enclosed system (Winters et al. 1995). RPE is the minimum potential energy that can be obtained through an adiabatic redistribution of the water masses. To compute RPE, we use the probability density function approach introduced by Tseng and Ferziger (2001). The RPE curves (Fig. 4) clearly show more mixing with increasing (decreasing) Froude number (density jump across the interface), as suspected on the basis of Fig. 3.

3) Sampling with passive tracers:

Passive tracer is released as a slab that is 50 m deep (approximately the depth of the mixed layer), 9 km long (just short of periodic domain boundaries, in order to allow for tilting by vertical shear), 150 m wide along the middle of the domain, and at three depth levels (Fig. 5). While this is somewhat crude, it is just the first step. The main problem with the tracer release is the resolution requirement to get close to oceanic conditions. Given the variable horizontal mesh of 16-57 m, the initial position is centered along an inter-element boundary where the mesh is finest. The background diffusivity is set much smaller than that for density, which is assumed to be controlled by the temperature field. The tracer field is released with the same initial condition every 15 days and the state of concentration at the end of the integrations is shown in Fig. 5. Some 3D type mixing during the initial phases and exponential stretching along the boundaries of turbulent structures is what can be readily identified, but the longer term distribution of the tracer field in the surface mixed layer is generally patchy and seems very complex. The differences between the distributions at the end of different phases could be more telling than the individual states. If the front starts showing major gaps of tracer concentration and the tracer is found large distance from the front, then this would be indicative of cross-frontal exchange.

4) Sampling with Lagrangian particles:

Synthetic Lagrangian particles are released in a 2 km by 5 km box in triplets that are 0.5 m to 20 m apart. Particles are released near the surface (5 m), near the mixed layer base (50 m) and below the mixed layer base (100 m). A total of 1875 triplets (5675 particles) are advected online using the fully 3D velocity field (Fig. 6). These launches are repeated for the different dynamical phases of the instability growth.

Of particular interest is relative (or two particle) dispersion, or the mean-square particle separation which is closely tied to scalar mixing processes. Relative dispersion is seen to fit in between the ballistic and Richardson regimes for times longer than a day, while relative dispersion at different levels is separated by how long the exponential regime lasts (Fig. 7a). Sub-sampling, targeted sampling of features, as well quantification of relative dispersion using the finite scale Lyapunov exponents are in progress.

Another novel use of the Lagrangian sampling would be through their vertical displacement. Fig. 7b indicates that rms vertical displacements can be emploted in order to quantify the differences in dynamical phases. For instance, during the restratification phase, we get a reduction in the rms vertical displacement with respect to the time at the onset of mixed layer instability.

5) Preliminary conclusions:

LES of mixed layer instability reveals sub-surface vortices at the mixed-layer base which seem to be an under-emphasized feature of this problem. I also find that weak fronts are characterized by mixing, while strong fronts are characterized by stirring via coherent eddies. Surface tracer sampling seems very challenging as it is quickly diluted by exponential stretching in regions of strain. Lagrangian sampling appears quite promising as it leads to several well-defined metrics such as relative dispersion and finite-scale Lyapunov exponents, which are closely related to the objectives of Lateral Mixing DRI. In particular, cluster launches in pairs or triplets may allow how transport processes change as a function of spatial scale, while fully 3D Lagrangian drifters can be used to identify the transition to restratification phase.

IMPACT/APPLICATIONS

The scales considered in this project represent the range of scale of navy operations and thus anomalous currents and perturbations in the acoustic and optical environment that can affect a variety of navy operations. Understanding the motion in this range of scales is therefore critical to help improve the predictive capability of the existing Navy models.

REFERENCES

Fischer, P.F., 1997: An overlapping Schwarz method for spectral element solution of the incompressible Navier-Stokes equations. J. Comp. Phys., 133, 84-101.

Özgökmen, T.M., T. Iliescu and P. Fischer, 2009a: Large eddy simulation of stratified mixing in a three-dimensional lock-exchange system. Ocean Modelling, 26, 134-155.

Özgökmen, T.M., T. Iliescu and P. Fischer, 2009b: Reynolds number dependence of mixing in a lock-exchange system from direct numerical and large eddy simulations. Ocean Modelling, 30, 190-206.

Tseng, Y. and J. Ferziger, 2001: Mixing and available potential energy in stratified flows. Phys. Fluids, 13, 1281-1293.

Winters, K.B., P.N. Lombard, J.J. Riley, and E.A. D'Asaro, 1995: Available potential energy and mixing in density-stratified fluids. J. Fluid Mech., 289, 115-128.

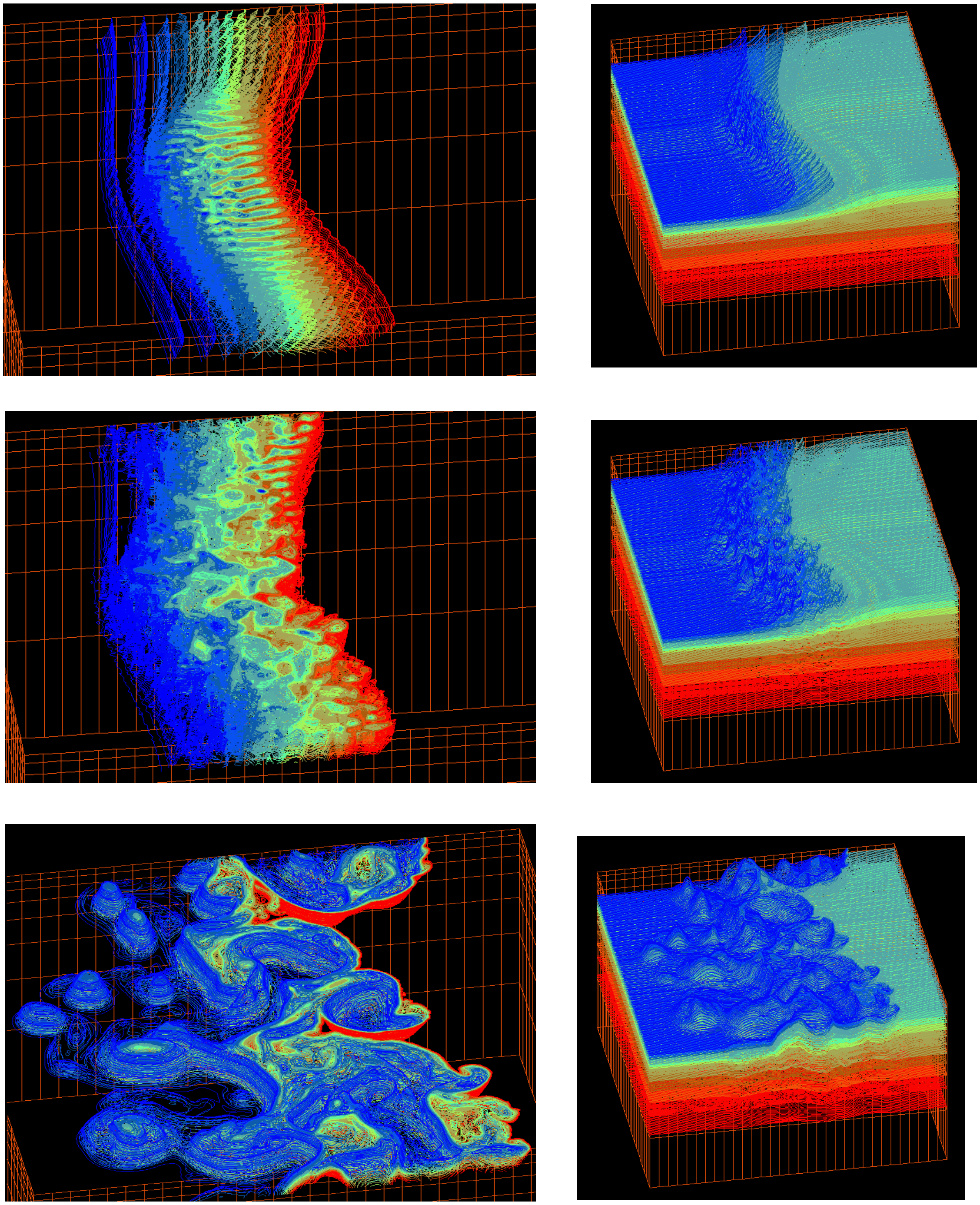

Fig. 1: (Upper panel) Initial interleaving across the density interface via inertial and symmetric instabilities. Right column shows the entire computational domain while left column focuses on the density interface. (Middle panel) Growth of larger scale features via backward energy cascade. (Lower panel) Destruction of the density interface during the restratification phase.

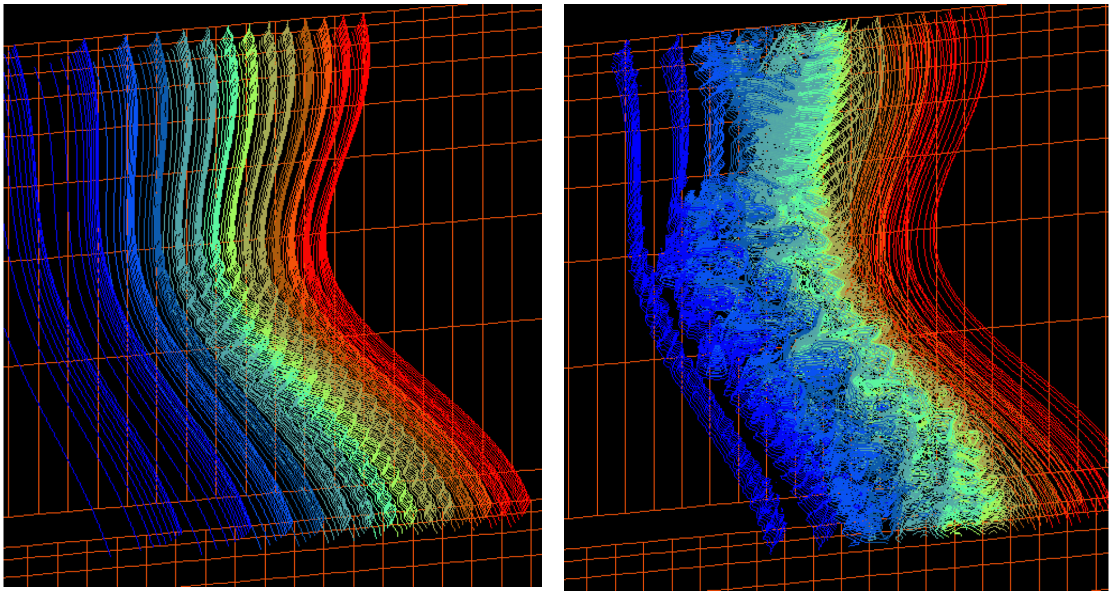

Fig. 2: Sensitivity of the growth of mixed layer instability to numerical dissipation. (Left panel) $Re=10^4$, (right panel) $Re=10^6$.

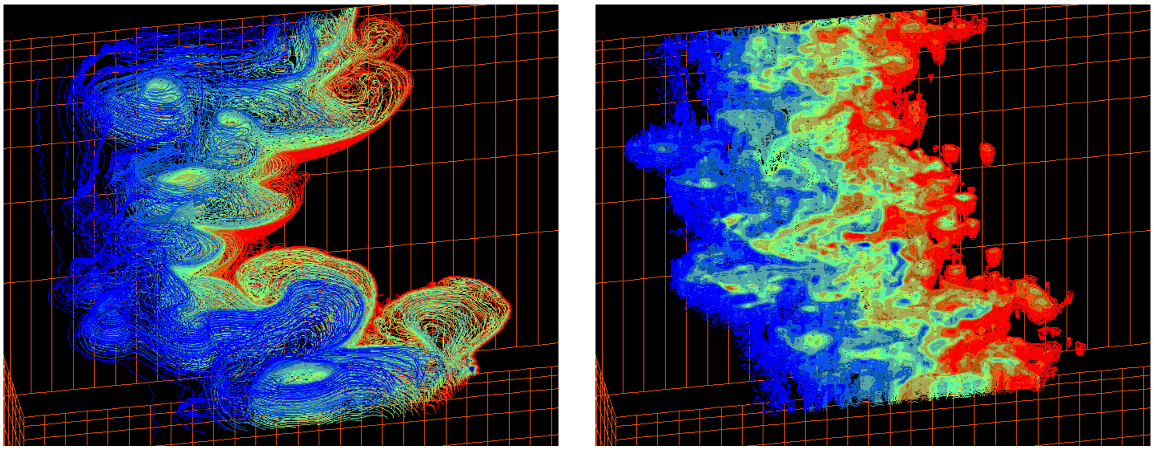

Fig. 3: Sensitivity of mixed layer instability to the initial density gradient across the interface. (Left panel) A case of large density difference leading to coherent structures and stirring. (Right panel) Smaller density difference causing mixing as indicated by enhancement of intermediate density classes (shown in yellow and green).

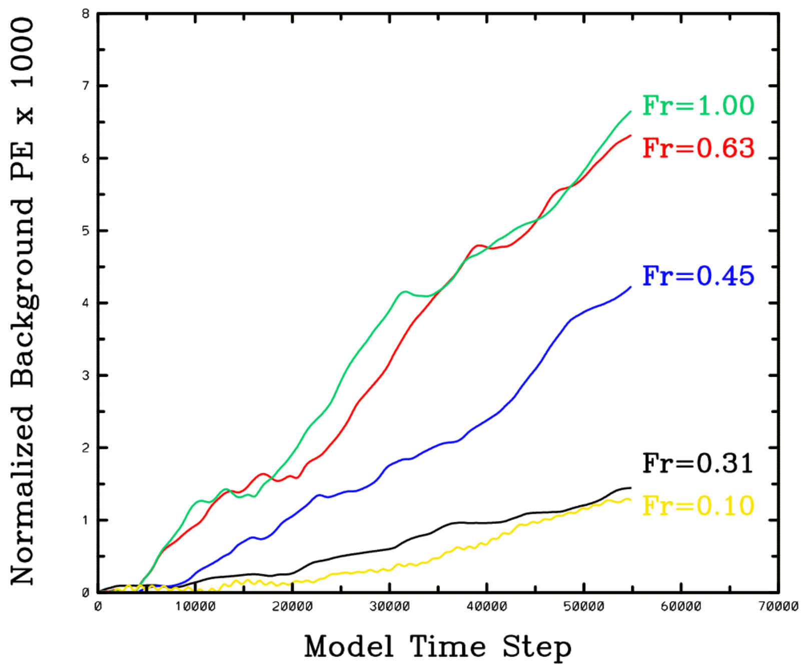

Fig. 4: Time evolution of background potential energy from experiments with different initial density difference, quantified by the Froude number.

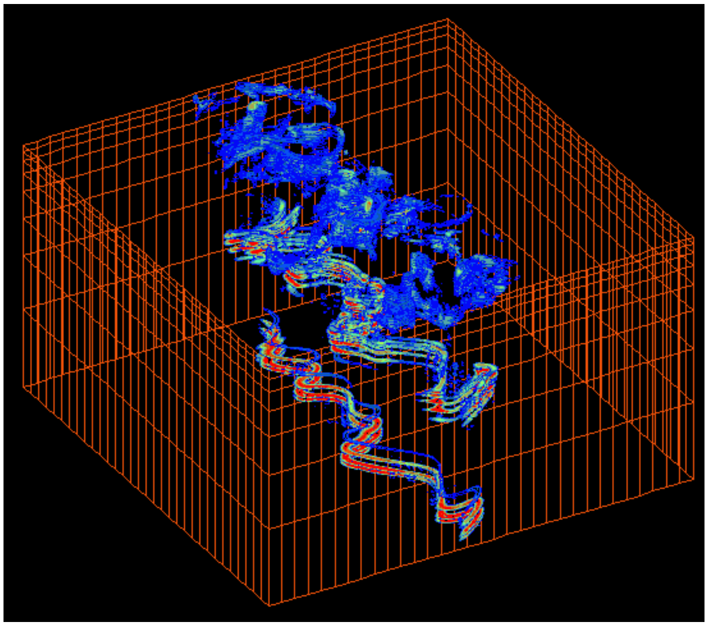

Fig. 5: Snapshot of the tracer field (for non-dimensional concentration > 0.1) at 10 days after being released at three depth levels during the mixed layer instability.

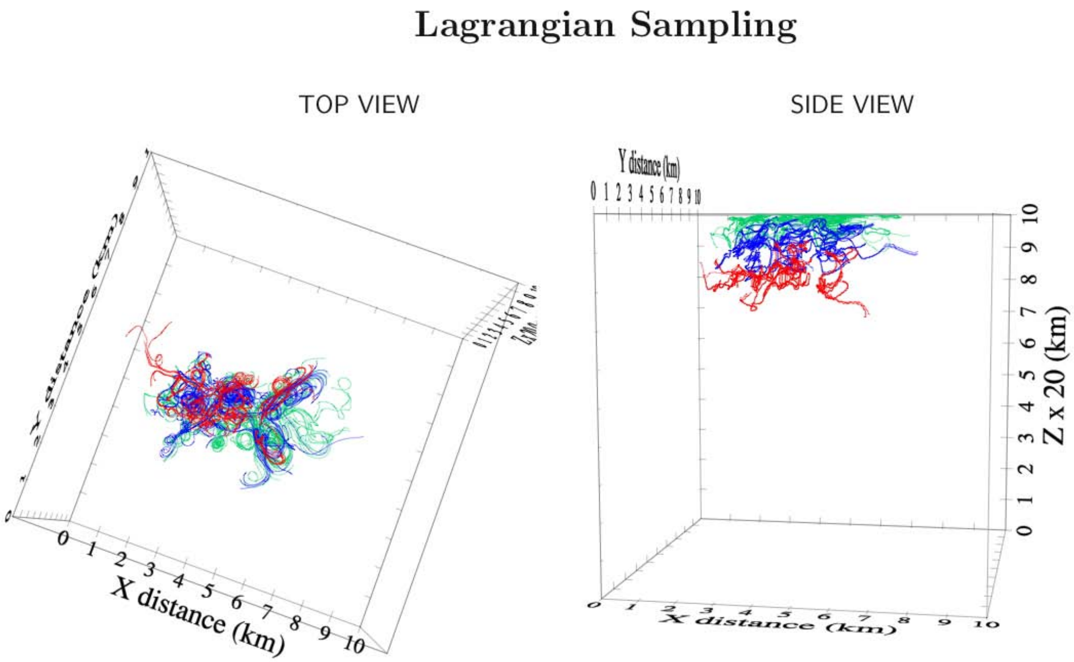

Fig. 6: Sample Lagrangian particle trajectories launched at different depth levels; 5 m: green, 50 m: blue and 100 m: red.

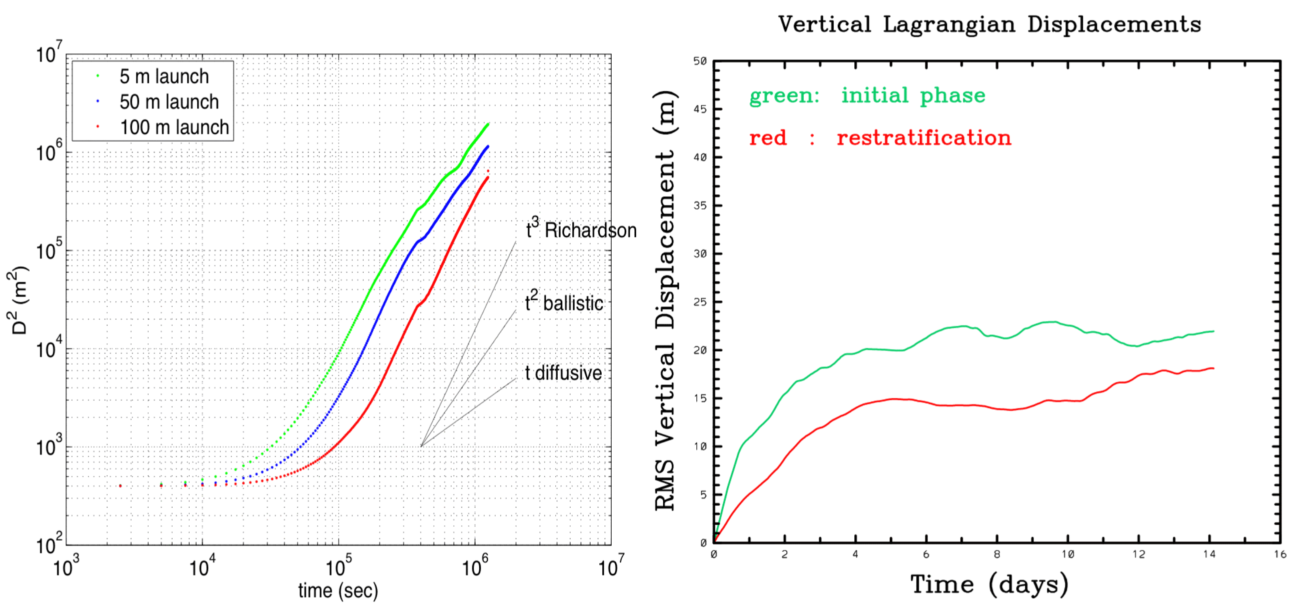

Fig. 7: (Left panel) Relative dispersion from particles launched at three depth levels. (Right panel) Rms vertical displacement from particles during different dynamical phases of the mixed layer instability.